The physics behind the rainbow©

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A rainbow is a beautiful and often awe-inspiring phenomenon that is familiar to most everyone. However, when you look into the sky after a summer thunderstorm, you will sometimes see a rainbow and sometimes not. It's not that the rainbow isn't there in some sense, as long as the sun is shining of course, but that you the observer may not be in the right location relative to the sun's and the thunderstorm's position. Seeing a rainbow is a matter of circumstance or coincidence but one need not rely on coincidence when creating a rainbow on demand, such as was done over the Bemis Center. The physics behind the formation of rainbows is well known and was described by the French philosopher and mathematician Rene Descartes nearly 400 years ago. An understanding of the physics of rainbows allows us to determine preferred viewing areas as well as understand some details of the rainbow display.

Visible light is part of an electromagnetic spectrum that ranges from very short wavelength, high-energy gamma and x-rays, through the ultraviolet, visible and infrared, and into the microwave and radio portion of the spectrum. The output of the sun is primarily within the visible portion of the spectrum and consists of a full pallet of colors, the combination of which we perceive as white light.

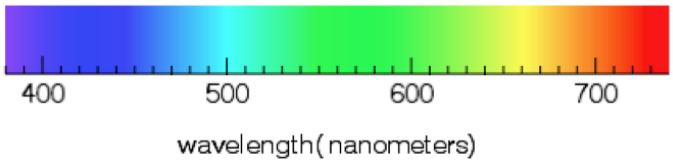

Figure 1: The visible spectrum of light.

Figure 1 shows the colors that comprise the visible portion of the spectrum ranging from violet through blue, green, yellow, orange and

red. The peak intensity of sunlight is concentrated in the green portion of the spectrum, with a wavelength of around 570 namometers (.00057 millimeters) and with lesser intensity at the shorter and longer wavelengths.

The speed of light in a transparent medium such as glass or water depends a property of the medium referred to as the refractive index. The refractive index determines how much the speed of light is reduced in the medium relative to the value in a vacuum. If ray of light is incident on a surface of glass at an angle, the variation of the speed of light across the interface results in the ray being bent as it passes through and the angle of the bending depends on the wavelength. The shorter wavelengths of light are bent at a larger angles with the result being that the beam of white light is separated into a spectrum of colors as it passes through the medium.

If visible light is incident on a water drop there is another factor that comes into play in addition to the refraction described above. As a ray of light enters a drop, some of the light passes through the back of the drop and some is reflected internally from the back of the drop. The result of this reflection and the separation of colors due to the refraction is that the color spectrum is returned back in the direction from which it came. There is a characteristic bending angle of about 42^o, with the exact angle depending on the wavelength of the light. The result is a band of color at the characteristic angle relative to the direction of the sunlight. The result of this internal reflection and the separation of wavelengths shown in Figure 2[1].

Figure 2 shows a cross-section of the path of a ray, but in reality light being reflected from the back of the drop is reflected in a 360^o arc, resulting in a ring of color emanating from the drop as is shown in Figure 3. When an observer views a

[1] It is possible for there to be a second internal reflection and a much less intense beam of light exiting the drop at an angle of around 53^o .This produces a dimmer secondary rainbow is some circumstances.

collection of drops, like the ones resulting from the rain shaft of a thunderstorm or

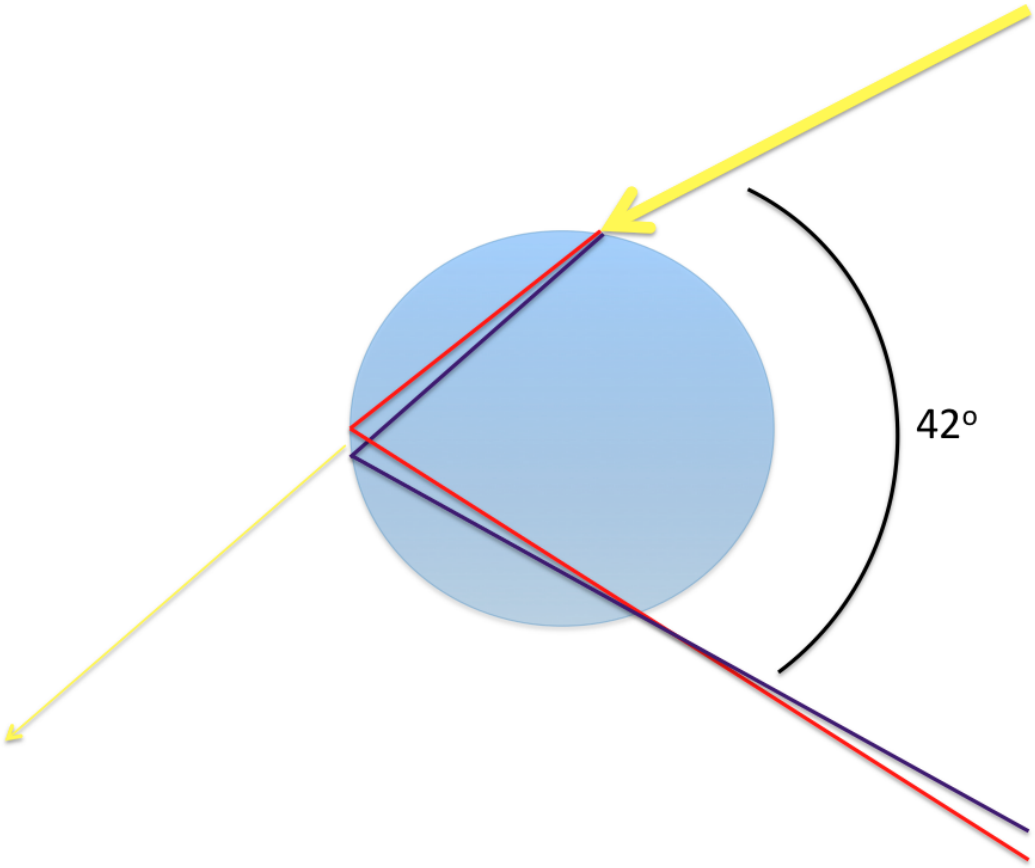

Figure 2: Refraction and total internal reflection of light inside a water drop.

from a stream of water sprayed into the air, he or she sees the light coming from the bottom of the drops overhead, from the right side of the drops to the right, etc. Each of these beams converge at the location of the observer, and the collection of drops for which the rays pass through the observer's location define the arc of the rainbow. There is a unique collection of drops for which the rays will pass through a given point, so each observer sees rays from a different set of drops, that is, his or her own personal rainbow. Also, if the observer were to move, a different collection of drops will determine the arc of the rainbow. Simply stated, the rainbow moves with the observer, which is why it's never possible to get to the end of the rainbow.

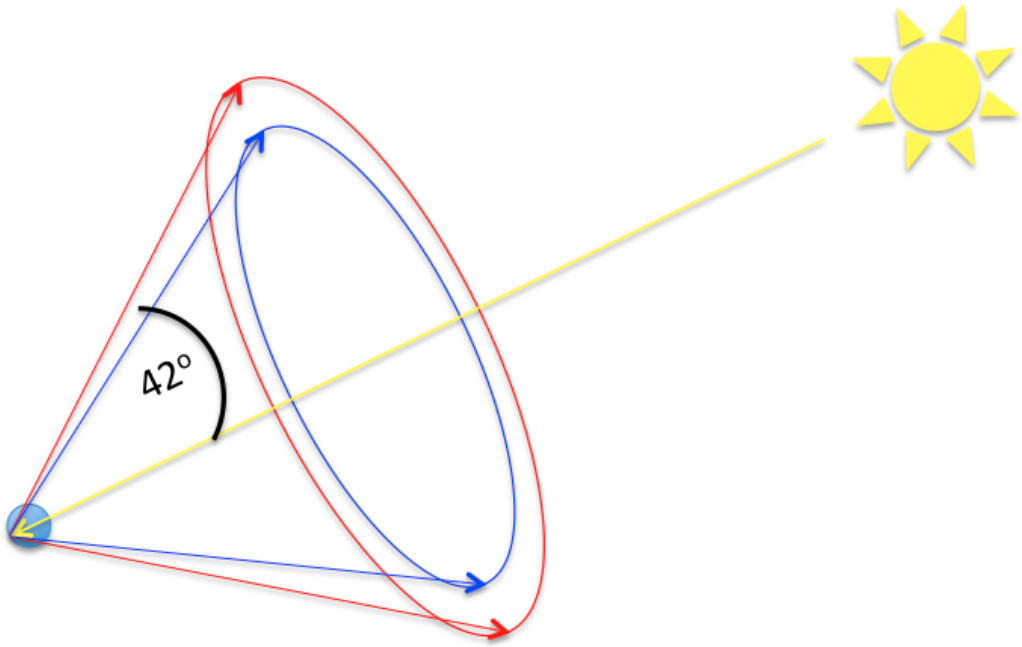

Figure 3: Three dimensional refraction and reflection of sunlight from a drop.

The incident sun angle defines a collection of rays that would pass through a given point and these rays define a cone of revolution around the incident sun angle, as is shown in Figure 4. Water drops at any point on this cone will produce a reflected spectrum, but since the water that produces rainbows typically occurs in vertical sheets, i.e. from thunderstorms or sprayed into the air from nozzle on the roof of a building, a vertical 'slice' of the cone is illuminated. This cone is referred to as a 'conic section'. The arc of a rainbow is in principle a full circle, but the portion of the circle that is beneath ground is obviously not visible to an observer at the surface. The existence of this cone of revolution means that at any time, as long as the sun is shining, there exists a virtual rainbow defined by the incoming sun angle. All that needs to happen is for water to be placed onto the cone, and the observer will see a rainbow.

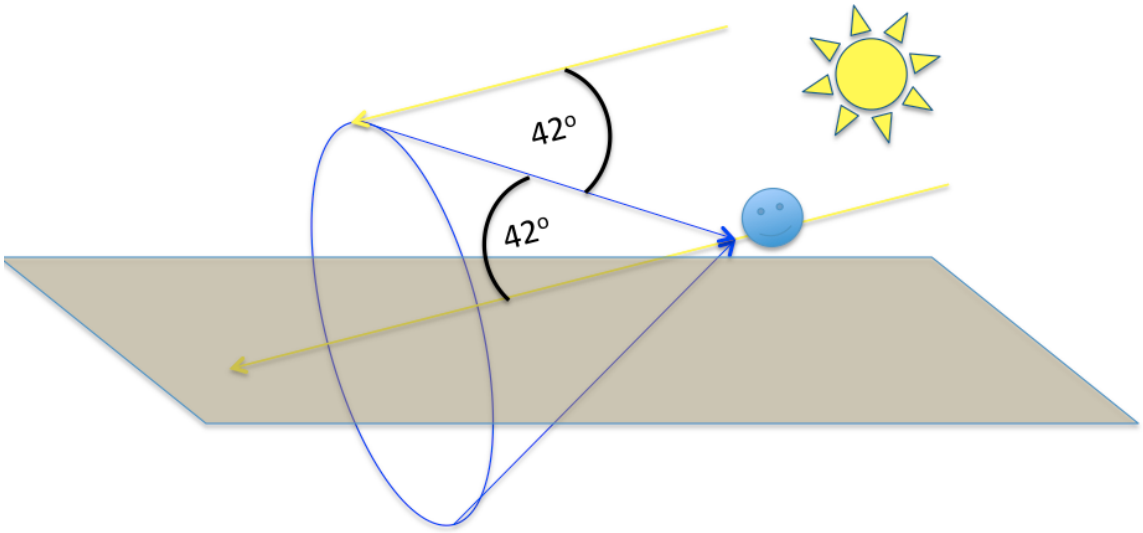

Figure 4: Cone of revolution produced by a rays extending from a collection of virtual drops and converging at the location of an observer

The details of the rainbow optics described above can be used to predict the location of the convergence point of the rays as is illustrated in Figure 4. The angle of the sun is given in terms of an elevation above the horizon. If there is a stream of water extending up from a point, say the roof of a building, projecting back from the top of the stream at a 42^o angle relative to the sun angle defines the point for which a ray strikes the surface and hence an outer range where a rainbow will be visible (see Figure 5a). Likewise, the bottom of the stream will define an inner range for which the rainbow can be seen. If the height of the top of the stream and the height of the rooftop, or equivalently the bottom of the stream, are known we can calculate the inner and outer distances using the formulae given below.

$$\text{Outer range} = \frac{\text{stream height}}{\tan(42^o - \text{sun angle})} \qquad \text{Inner range} = \frac{\text{roof height}}{\tan(42^o - \text{sun angle})}$$

An observer standing anywhere between the inner and outer range will see a color band somewhere along the stream.

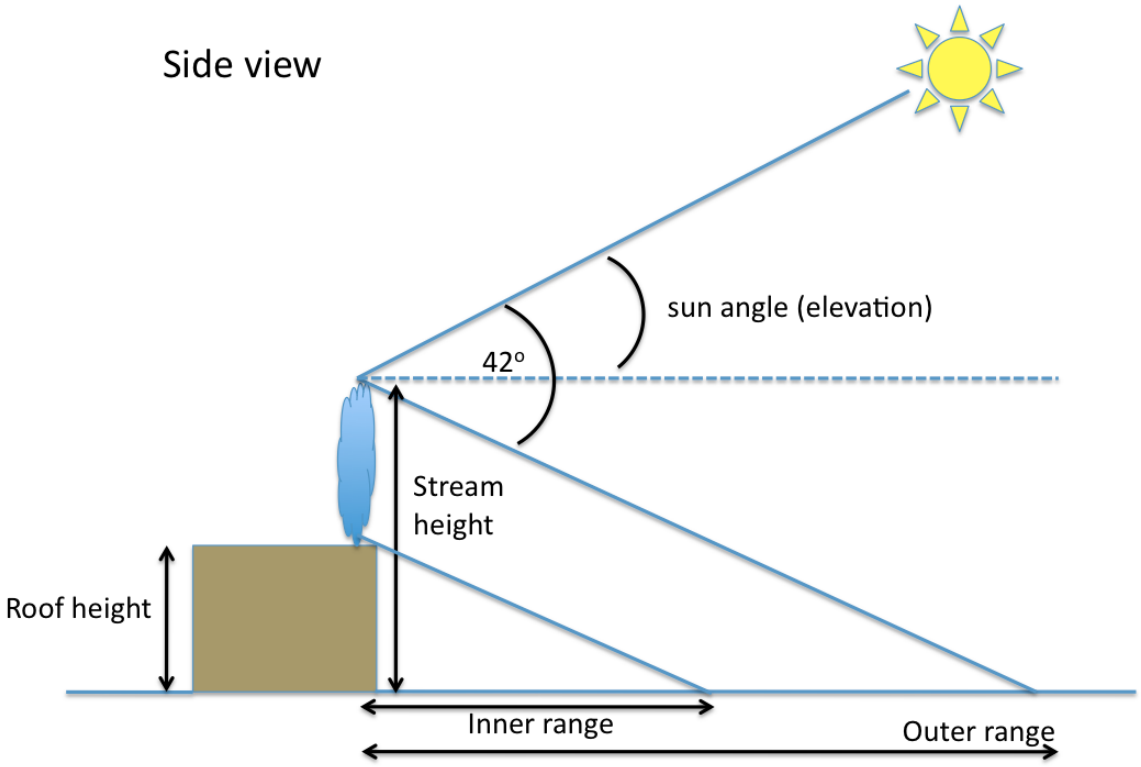

Figure 5a: Side view defining the inner and outer view ranges for a water stream extending from the top of a building.

As the sun angle changes so does the location of the range where the rainbow is visible. Early in the morning and late in the evening, the range is located relatively close to the stream, but as the sun rises in the morning, the ranges move to larger distances. While it would still be possible to produce a rainbow display, the farther the ranges are located from the building, the greater the chances that buildings or other obstacles will obscure the view. Once the sun higher that 42° above the horizon, the rainbow is no longer visible from the ground as the rays no longer intersect. This occurs somewhere around 10:00am local time during the summer. As the sun moves across the sky and then starts to set, the rainbow will becomes visible

once the sun is less than 42° above the horizon. As a result, the rainbow displays are only possible during the morning and evening hours during the summer.

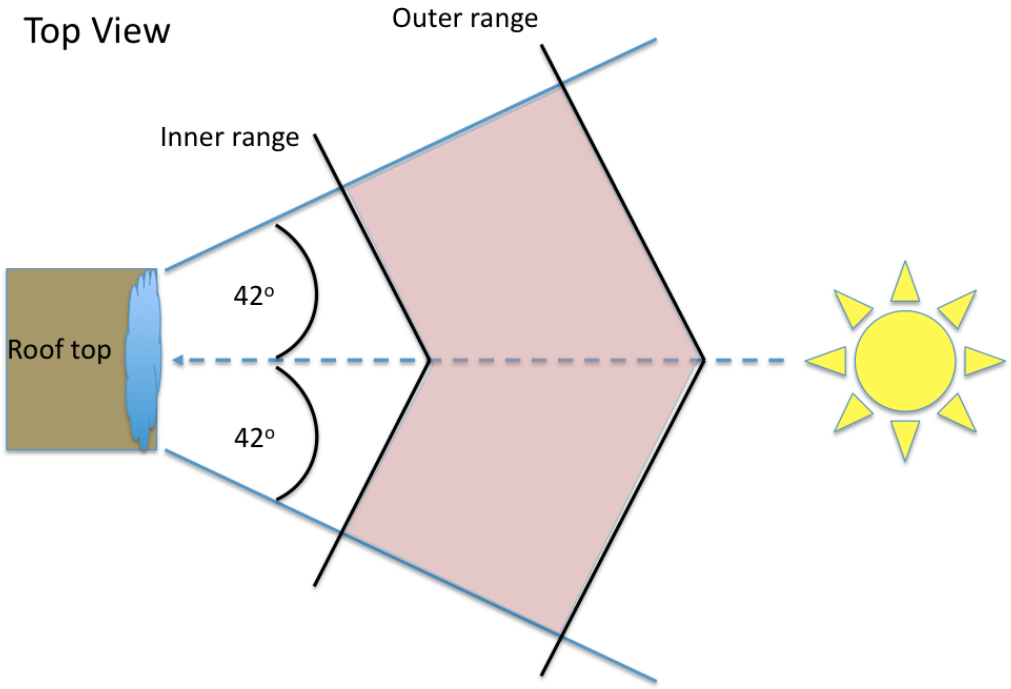

Figure 5b: Top view of the optimal field of view

The cone of revolution around the incident sun angle defines left and right extent of the field of view as is shown in Figure 5b. An observer standing anywhere inside the shaded area will see a rainbow somewhere on the stream. A specific example of the fields of view is shown in Figure 6, which gives the field of view corresponding to 9:00am (red) and 8:00pm (yellow) for early July. The fields of view change with time of day and also day of year as the sun rises further to the north and the sun angle becomes shallower as the season progresses. A series of charts of the kind shown in Figure 6 were prepared for each week of the exhibit to guide viewers toward optimal locations for viewing.

The fields of view shown in Figure 6 were calculated by assuming that the stream is centered on the building and on the edge closest to the viewer. That is, the east side

Figure 6: Sample fields of view calculated for July 2 at 9:00am (red) and 8:00pm (yellow). The Bemis Center for Contemporary Arts is in the center.

during the morning and the west side in the evening. Depending on specific wind conditions of the day there could be water drops dispersed over a fairly large area around the building and viewers outside the optimal field of view would see a rainbow, or parts of one. This was typically true for viewers to the left or right of the field of view. Someone standing outside of the outer range would only see a stream of water.

The internal reflection and bending of the light rays that produce the rainbow is an example of scattering of light. This particular optical effect works best when the scatterers, the drops, are large enough that they act as prisms or lenses. Scattering of light by large objects is referred to as geometric optics. If the scatterers are very small, like say air molecules, then the behavior is different from that shown in Figure 2 and that produces a rainbow.

The scattering of light by objects that have a size smaller than the wavelength of the light, such as air molecules, is referred to as Rayleigh scattering (named after British physicist Lord Rayleigh). For Rayleigh scattering, the light is scattered in all directions rather than in the coherent beam produced by geometric optics. However, the efficiency of the scattering depends on the wavelength of the light, with the shorter wavelengths being scattered more efficiently. That is, the purple and blue colors of visible light are scattered most efficiently and red the least. There is relatively little purple in the incident solar radiation so it's primarily the blue light that is scattered while the rest of the colors of the spectrum pass through. An observer looking into the sky away from the direction of the sun sees the scattered blue light. This is why the daytime sky appears blue.

The Rayleigh scattering is wavelength dependent, with the scattering being most efficient for the shorter wavelengths, but the scattering still occurs for all wavelengths. If the sunlight has to pass through a very thick layer of atmosphere, which happens at sunset, then only the long wavelength part of the spectrum passes through unimpeded while the shorter wavelength parts are scattered. An observer looking at the setting sun will see only the longest wavelength part of the spectrum. This is why the sun and sky appear red at sunset.

If the objects responsible for the scattering are comparable in size to the wavelength of light, then the interaction is governed by the principles of Mie scattering, named after the German physicist Gustav Mie. Mie scattering still occurs in more or less all directions, similar to Rayleigh scattering, but is no longer strongly dependent on wavelength. That is, all colors of the spectrum are scattered with a comparable efficiency. The incident white light is scattered in all directions without the color separation produced by Rayleigh scattering. Dust particles and particularly those that have absorbed water vapor and increased in size are efficient as Mie scatterers. The fact that white light is scattered in all directions, independent of wavelength, is why the sky appears milky or grey during periods of high humidity. Cloud drops,

that is drops that are too small to fall out as rain, are also efficient Mie scatterers, which is why fair weather cumulus clouds appear white.

In order to provide as large a field of view for the rainbow display as possible, it was necessary to maximize the height of the water stream while keeping the flow rate as low as possible to conserve water. Ideally, one would like to produce drops that are within the size range for geometric optics to be effective in order to maximize the rainbow effect. A size selection of this type isn't possible in any sort of practical way, and the stream was composed of a collection of drops of a wide range of sizes. The ascending part of the stream is more or less continuous, with drops of a range of sizes being sheared from the edge of the stream. As the stream reaches its maximum height and begins to fall, the water separates into drops of a wide variety of sizes. Large drops are influenced more by turbulence and wind resistance and break into smaller drops, further complicating situation. As the collection of drops begins to fall, the drops separate by size with the larger ones being able to fall past the turbulent updrafts while the smaller ones are carried up and away. As a result, there is a selective distribution of water drops with the ones being efficient in producing the geometric optics effects that separate the visible light into a color spectrum falling back down toward the surface, resulting in the rainbow.

Some very good examples of the issues related to drop size were captured by a variety of photographers during the course of the exhibit. The separation of colors due to the geometric optics is much more efficient in places that are composed of the larger falling drops, that is in between the water streams, as is seen in Figure 7a. Sunlight is striking the ascending portion of the stream, but since it is composed of the full spectrum of drops the Mie scattering dominates and the colors are muted with the stream appearing predominantly white. The selective descent of the larger drops into the regions between the streams provides for color separation in those areas. However, owing to the finite width of the ascending part of the stream there is still a combination of Mie scattering and geometric optics in this region. This results in the colors being somewhat muted. In areas that are farther from the water

streams there tend to be only the larger drops present and the separation of colors is more complete with little of the 'contamination' by scattered white light, as is seen in Figure 7b. This figure also shows evidence of a secondary rainbow that is produced by two internal reflections within the drop.

Figure 7a,b: Sample rainbow displays showing the effect of drop size on the color intensity

Geometric optics relies on the light entering the drops in the form of a parallel beam. In this way, the coherent bending and internal reflection of light can occur. If there is anything between the sun and the water stream (or a rain shaft of a thunderstorm for that matter), that can make the incoming sunlight diffuse, the separation of the light into a color spectrum is supressed. A layer of high thin cirrus clouds, composed primarily of very small ice crystals, while not limiting the intensity of the incoming solar radiation, would cause the light to become diffuse through Mie scattering and greatly limit the intensity of the rainbow.

This section provides an overview of the physics behind the production of rainbows. A knowledge of geometric optics, and particularly the changes in the angle of the incoming light, along with some basic trigonometry allows for the determination of the inner and outer ranges as well as the horizontal spread of the field of view. Some of the subtleties of the appearance of the rainbow can be understood in terms of the different response of light to drops of various sizes.

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